Angular momentum of betatron radiation from laser wakefield accelerators

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In addition to linear momentum, light also carries angular momentum. Just like any particle, light can also carry momentum. Circular polarisation

Transverse vector field of a circularly polarised Gaussian laser

Assume two values: right-handed (spin 1) or left handed (spin -1) circularly polarised

Conservation of angular momentum

Beth's experiments showed that light's angular momentum exerted a torque on matter:

Can a circularly polarised laser driver exert a torque on relativistic electrons which in turn will produce circularly polarised x-rays?

Allen et al PRA 1992
The angular momentum of light can exert a torque on relativistic plasma electrons

3D OSIRIS simulation with a mixture of Helium with Nitrogen

- ~99%He + ~1%N
  \((10^{18}-10^{19} \text{ cm}^{-3})\)
- circularly polarised laser
  \((I\sim10^{18} \text{ W/cm}^2)\)

The laser modulates the outer Nitrogen shell electrons

*J.L. Shaw et al PPCF 56, 084006 (2014)
The angular momentum of light can exert a torque on relativistic plasma electrons.

3D OSIRIS simulation with a mixture of Helium with Nitrogen

*J.L. Shaw et al PPCF 56, 084006 (2014)

DLA leads to helical bunch currents and trajectories

e-\text{'s respond to laser polarisation

Helical spatial structure

Clockwise azimuthal currents

Helical trajectories

Reversed polarisation direction

Radiated energy

\[ E_{\text{pixel}} = \frac{e^2}{4\pi c} \sum_p \int \left| \frac{n \times [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^5 R^2} \right|^2 S_{\text{pixel}} \, dt \]

J.D. Jackson, Classical Electrodynamics

Spectrum calculation

**general**

\[ \frac{d^2 I(\omega)}{d\omega d\Omega} = \frac{e^2}{4\pi c} \left| \sum_p \int_{-\infty}^{+\infty} \frac{n \times [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^2} \exp[i\omega(t + R/c)] \, dt \right|^2 \]

**far-field**

\[ \frac{d^2 I(\omega)}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c^5} \left| \sum_p \int_{-\infty}^{+\infty} n \times (n \times \beta) \exp[i\omega(t - n \cdot r/c)] \, dt \right|^2 \]

J. D. Jackson, Classical Electrodynamics

J. L. Martins et al., Proc. SPIE 7359, 73590V (2009);
Circularly polarised x-rays

Degree of circular polarization

-0.4 -0.0 0.4

right hand circular polarisation

left hand circular polarisation

Polarisation control

\[ \phi = \pi / 2 \]

\[ \phi = \pi / 4 \]

\[ \phi = 0 \]

\[ \phi = -\pi / 2 \]

We may be able to control the spin angular momentum of x-ray photons for high energy density science.

Betatron x-ray polarisation can be adjusted by driver polarisation.
Circular polarisation

Transverse vector field of a circularly polarised Gaussian laser

Local rotation of the electric and magnetic field vectors. Two possible values: left and right handed rotation

Orbital angular momentum

Helical/spiral phase fronts: photons have an azimuthal velocity component. New intrinsic degree of freedom which can take nearly any integer value.

The angular momentum of light

Just as electrons and other particles, photons can also have angular momentum

J.L. Martins | AAC, National Harbor MD USA | August 2, 2016
Phase calculation

Decomposition into clockwise/counter-clockwise components

\[ \vec{E}_+ = \vec{E} \cdot \vec{e}_+ \quad \quad \vec{E}_- = \vec{E} \cdot \vec{e}_- \]

\[ \vec{e}_+ = \frac{\vec{e}_x + i\vec{e}_y}{\sqrt{2}} \quad \quad \vec{e}_- = \frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}} \]

phase calculation

\[ \Phi_+ = \tan^{-1} \left( \frac{\text{Im}(E_+)}{\text{Re}(E_+)} \right) \quad \Phi_- = \tan^{-1} \left( \frac{\text{Im}(E_-)}{\text{Re}(E_-)} \right) \]

* S. Sasaki and I. McNulty, PRL 100, 124801 (2008)
Helical wiggler harmonics

Energy

Spectrum

\[
\omega_1 \approx 4000 \omega_0 \\
\omega_2 \approx 8000 \omega_0 \\
\omega_3 \approx 12000 \omega_0
\]

\[
\lambda_0 = \frac{2\pi \bar{\beta} c}{\omega_0}
\]

\[
\bar{\beta} = \text{average long. vel.}
\]

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No OAM in first harmonic*

Spectrum at $\omega = 4000 \omega_0$

* S. Sasaki and L. McNulty, PRL 100, 124801 (2008)
Single phase discontinuity associated with OAM mode $\ell = 1$*

Spectrum at $\omega = 8000 \omega_0$

- Harmonics 3, 4 and 5
- Zoom region

* S. Sasaki and I. McNulty, PRL 100, 124801 (2008)
Two phase discontinuities associated with OAM mode $\ell = 2^*$  

Spectrum at $\omega = 12000 \omega_0$  

Zoom region  

Harmonics 4, 5 and 6  

Spectrum zoom  

Phase zoom  

* S. Sasaki and I. McNulty, PRL 100, 124801 (2008)
Phase shows OAM mode $\ell = 2$ at laser frequency.

Spectrum slice at $\omega \sim \omega_0$

Phase

2 discontinuities crossed

in preparation (2016)
Conclusions & Future Work

Implemented phase and polarisation calculation in jRad

- Fully characterise radiation angular momentum polarisation
- Polarisation and orbital angular momentum in a variety of plasma and astrophysical settings, including laser and plasma wakefield accelerators

Production of circularly polarised betatron x-rays in the LWFA

- Polarisation controlled by changing the laser driver polarisation
- Ongoing work to evaluate the conditions to enhance polarisation level

Production of betatron radiation with orbital angular momentum

- jRad calculations in agreement with theoretical predictions
- Results indicate that betatron radiation can carry OAM