Understanding perturbations to nonlinear integrable optics in IOTA


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Outline

• Integrable optics at IOTA

• Synergia 2.1 for beam dynamics simulations

• Nonlinear dynamics in the zero current limit
  • Higher order terms in the nonlinear Hamiltonian
  • Variation with $H_0$ and emittance
  • Correlations between 1st and 2nd invariant

• Chromaticity correction for integrable machines
  • Nonlinear chromaticity and dispersion in IOTA
  • Paired correction for improved momentum acceptance

• Nonlinear dynamics with space charge and future work
Motivation

• Meeting the goals of the Intensity Frontier requires support for MW-class hadron beams
  – Science goals (BSM) include matter-antimatter asymmetry, neutrino oscillations, muon decay

• High beam power presents significant dynamics challenges
  – Space charge driven tune shift drives resonance crossings
  – Bunch oscillations drive particles to large amplitudes - so called beam halo - and increase losses
    • Machine protection requires $< 1 \text{ W/m} (< 0.1\% \text{ losses})$

• Accelerators recoup stability through introducing external (perturbative) nonlinearities
  – I.e. Octupoles generate tune spread with amplitude to damp resonances - nonlinear decoherence

• Most nonlinearities do not preserve regular, periodic motion in the transverse plane! These systems are non-integrable.
Integrable Optics and the IOTA lattice

- Experimental initiative to test nonlinear integrable optics
  - Danilov & Nagaitsev “Nonlinear accelerator lattices with one and two analytic invariants,” PRSTAB 13, 084002 (2010)
- Use of special nonlinear magnet can result in a 2nd invariant of motion, completely integrable dynamics
  - Single particle trajectories are regular and bounded
  - Mitigate parametric resonances via nonlinear decoherence
  - Specific symmetries required:
    - $n\pi$ phase advance between NL inserts
    - $\beta_x(s) = \beta_y(s)$, $D(s)=0$ through underlying drift region
    - Potential is piecewise-constant in $s$

A. Romanov, “IOTA Optics Update,” presented at Fast/IOTA Scientific Workshop (Batavia, June, 2016);
Beam dynamics and space charge via Synergia 2.1

Synergia: A comprehensive accelerator beam dynamics package
http://web.fnal.gov/sites/synergia/SitePages/Synergia%20Home.aspx

Accelerator Simulation Group
James Amundson, Qiming Lu, Alexandru Macridin, Leo Michelotti, Chong Shik Park, (Panagiotis Spentzouris), Eric Stern and Timofey Zolkin

Computer time from INCITE

The ComPASS Project
High Performance Computing for Accelerator Design and Optimization
https://sharepoint.fnal.gov/sites/compass/SitePages/Home.aspx
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Higher order terms in the Nonlinear Hamiltonian

• Original analysis computed the single turn map for the ideal integrable optics lattice

• Expansion of the Hamiltonian lowest order correction to $H$ in $\nu_0$, the “tune-advance” through the nonlinear drift

\[
\mathcal{H} = \nu_0 \left\{ \frac{1}{2} (p^2 + x^2) + tV(\hat{x}, \hat{y}) \right\} + \\
\frac{1}{6} \nu_0^3 \left\{ \frac{1}{4} [\bar{p}^2 + \bar{x}^2, [\bar{p}^2 + \bar{x}^2, tV(\hat{x}, \hat{y})]] - \frac{t^2}{2} [V(\hat{x}, \hat{y}), [V(\hat{x}, \hat{y}), \bar{p}^2 + \bar{x}^2]] \right\} + O(\nu_0^5).
\]

• The leading order correction to the ideal Hamiltonian $H_0$ scales with $\nu_0^3$, thus implying a variation in $H$ from the first order estimate $H_0$

\[
\mathcal{H} = H_0 \nu_0 + H_2 \nu_0^3 + \ldots
\]
Variation in the first invariant - $H_0$

• Simulated a toy-model IOTA lattice, comprised of a nonlinear element followed by a corresponding 6x6 matrix representing a thin double-focusing lens.
  – Variations of the nonlinear element with different $\nu_0$ are calculated and scaled using a MADX script.
Variation in $H_0$ with increasing emittance

- Greater variation in $H_0$ with increasing emittance — coefficients in the expansion of the Hamiltonian vary with $\varepsilon^3$.

- i.e. For a NL segment with $\nu_0 = 0.3$, a KV distribution with $H_0 = 10$ mm-mrad demonstrates an average r.m.s. variation of 5% in calculated value of $H_0$. 

Higher order correlations between $H$ and $I$

- Turn by turn variation observed over many turns
  - Variations appear bounded and well behaved
- Suggestive of some surface relating to a new invariant
  - Computation requires nonlinear normal form analysis

Correlation between 1st and 2nd invariants - 10K turns

Correlation between 1st and 2nd invariants - 100K turns
Nonlinear chromaticity, dispersion in IOTA

- IOTA is a small ring with tight focusing.
  - Large phase advance yields large natural chromaticity
  - Dipole nonlinearities contribute significantly to focusing, further nonlinear chromatic effects
- Nonlinear dispersion complicates chromaticity correction

A single turn around IOTA with varying $\delta p$ illustrates nonlinear dispersion.
Chromatic correction perturbs integrability, acceptance

- Full chromatic correction strongly distorts particle motion through nonlinear insert
- Considerable beam loss and reduced momentum acceptance (<0.1%)
Chromaticity Correction for IOTA

• For realistic energy spreads, chromaticity significantly perturbs integrable motion

\[ \overline{H} = \overline{H}_0 + \Delta C \left( p_x^2 + x^2 - p_y^2 - y^2 \right) \]

• Work by Webb et al. presents a chromaticity correction scheme which may preserve integrability
  • Require equal horizontal and vertical chromaticities \( C_x = C_y \)
  • Sextupoles must be properly paired \( (\delta \phi = (2n + 1)\pi) \)

• Recover normalized Hamiltonian with adjustment to nonlinear strength parameter, \( t \)
  • Carefully placed sextupoles correct for linear chromaticity
  • Free to choose arrangement which maximizes aperture
  • Only one pair of sextupoles are required!
Proper correction improves acceptance

• Full chromatic correction introduces unwieldy sextupolar contributions to motion, particle loss
  – IOTA is sensitive to sextuple fields
  – NL inserts do not compensate for lost aperture

• Adjusted design with proper pairing of sextuples. $C_x=C_y$ fit with minimal sextuple strength - **30x lower strength**

• Particle loss reduced, dynamics improved

• Energy spread renders original invariant incorrect, but invariant motion should remain, can calculate through transformation:

$$l(x,t) \rightarrow l(x,t,C\delta)$$
Future Work

• Consider nonlinear optics in larger rings
  – Plans to simulate Fermilab Recycler ring as possible use case for nonlinear inserts
  – Evaluate performance at large beam power and tune depression with reduced ring nonlinearities

• Understand dynamics with space charge
  – Investigate single particle dynamics with low nonlinearity
  – Continue work with analytic “frozen” models, K-V and B-E
  – Calculate diffusion coefficient in perturbative regime

• Simulate wakefields in IOTA
  – Quadrupolar term along vertical axis may limit peak current
Conclusions

• Zero-current studies of IOTA performed to assess integrability
  1. Baseline performance evaluated for idealized ring
  2. Scaling laws demonstrated for realistic beam

• Careful design mitigates sensitivity to variations in momentum
  1. Nonlinear chromaticity and dispersion are difficulties
     1. Beam cooling and emittance reduction improves performance
     2. Reduction in tune, natural chromaticity, or nonlinear contributions to focusing improve integrability - favorable for large rings
  2. Proper chromaticity correction schemes improve acceptance while also reducing required sextuple strengths
  3. Integrability may be recovered accounting for sextuple strengths and linear dispersion

• Space charge compensation in IOTA requires detailed study with different distributions and solvers
Extras
Space charge compensation in IOTA

• Nonlinear insert is extremely sensitive to tune variations
  – Space charge introduces incoherent tune shift
  – Particles are “tune depressed” in both planes

• Compensate for linear component, and minimize incoherent component through choice of distribution
  – Generalized KV distribution no longer produces “linear” self forces, remains unstable
  – Waterbag and Semi-Gaussian distributions allow for smooth relaxation to proper matching

• Investigate evidence for diffusion in the invariants
  – Beam undergoes fast relaxation to a matched phase space
  – Oscillations about that distribution drive diffusive dynamics in the single particle invariants